# Time-like Baryon Form Factors and Dispersion Relations

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CENTRO FERMI

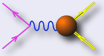
## Electromagnetic Interactions with Nucleons and Nuclei

EINN 2009

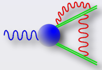
Milos Conference Center George Eliopoulos

Milos Island, September 28 - October 2, 2009

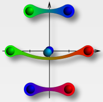
# Agenda



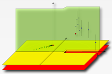
**Threshold behavior in $e^+e^- \to p\bar{p}$**



**Other charged baryons cross sections**



**$e^+e^- \to \Lambda\bar{\Lambda}, \Sigma^0\bar{\Sigma}^0, \Lambda\bar{\Sigma}^0$ puzzle**



**Space and time-like $G_E^p / G_M^p$ via DR's**

Bruno Touschek (1921, 1978) was an Austrian physicist, initiator of research on electron-positron colliders. He graduated from the University of Göttingen in 1946. He worked at the Max Planck Institute and at Glasgow. In 1952 he received the position of researcher at the National Laboratories of the Istituto Nazionale di Fisica Nucleare in Frascati.

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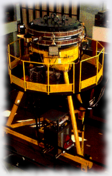
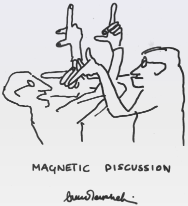


The first electron-positron collider was the ”Anello di Accumulazione” (AdA), built by Bruno Touschek in Frascati (Rome) in 1960.



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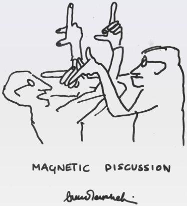
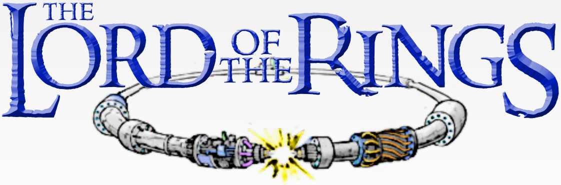
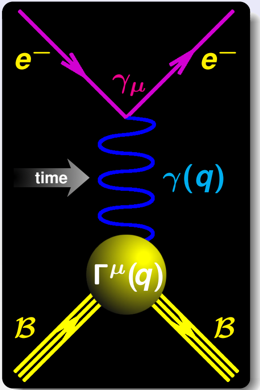


# Baryon Form Factors definition Space-like region ($q^2 < 0$)




- **Electromagnetic current** ($q = p' - p$)

$$J^\mu = e\overline{u}(p')\Gamma^\mu u(p) = e\overline{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M}F_2(q^2)\right]u(p)$$

- **Dirac and Pauli form factors $F_1$ and $F_2$ are real**
- **In the Breit frame**

$$\begin{cases} p = (E, -\vec{q}/2) \\ p' = (E, \vec{q}/2) \\ q = (0, \vec{q}) \end{cases} \qquad \begin{cases} \rho_q = J^0 = e\left[F_1 + \frac{q^2}{4M^2}F_2\right] \\ \vec{J}_q = e\,\overline{u}(p')\vec{\gamma}u(p)\,[F_1 + F_2] \end{cases}$$

- $2M\overline{u}(p')\gamma^\mu u(p) = \overline{u}(p')[(p+p')^\mu + i\sigma^{\mu\nu}q_\nu]u(p)$
- $\overline{u}(-\vec{p})u(\vec{p}) = E/M$
- $u^\dagger(-\vec{p})u(\vec{p}) = 1$

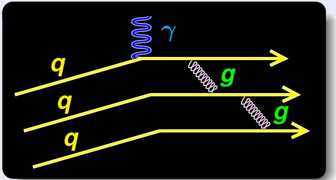
- **Sachs form factors**

$$G_E = F_1 + \frac{q^2}{4M^2}F_2$$
$$G_M = F_1 + F_2$$

- **Normalizations**

$$F_1(0) = Q_\mathcal{B} \qquad G_E(0) = Q_\mathcal{B}$$
$$F_2(0) = \kappa_\mathcal{B} \qquad G_M(0) = \mu_\mathcal{B}$$

# pQCD asymptotic behavior
# Space-like region




- **pQCD**: as $q^2 \to -\infty$, asymptotic behaviors of $F_1$ and $F_2$ must follow counting rules
- Quarks exchange gluons to distribute momentum

**Dirac form factor $F_1$**

- Non-spin flip
- Two gluon propagators
- $F_1(q^2) \underset{q^2 \to -\infty}{\sim} (-q^2)^{-2}$

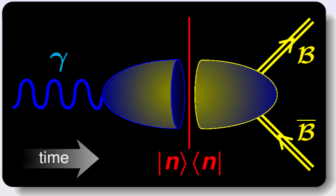
**Pauli form factor $F_2$**

- Spin flip
- Two gluon propagators
- $F_2(q^2) \underset{q^2 \to -\infty}{\sim} (-q^2)^{-3}$

**Sachs form factors $G_E$ and $G_M$**

- $G_{E,M}(q^2) \underset{q^2 \to -\infty}{\sim} (-q^2)^{-2}$
- Ratio: $\dfrac{G_E}{G_M} \underset{q^2 \to -\infty}{\sim}$ constant

# Baryon form factors
# Time-like region ($q^2 > 0$)




- **Crossing symmetry**:

$$\langle \mathcal{B}(p')|j^\mu|\mathcal{B}(p)\rangle \longrightarrow \langle \overline{\mathcal{B}}(p')\mathcal{B}(p)|j^\mu|0\rangle$$

- **Form factors are complex functions of $q^2$**

## Optical theorem

$$\mathrm{Im}\langle \overline{\mathcal{B}}(p')\mathcal{B}(p)|j^\mu|0\rangle \sim \sum_n \langle \overline{\mathcal{B}}(p')\mathcal{B}(p)|j^\mu|n\rangle\langle n|j^\mu|0\rangle \implies \begin{cases} \mathrm{Im}F_{1,2} \neq 0 \\ \text{for } q^2 > 4M_\pi^2 \end{cases}$$

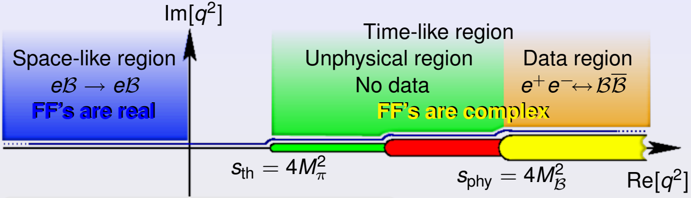
$|n\rangle$ are on-shell intermediate states: $2\pi, 3\pi, 4\pi, \ldots$

## Time-like asymptotic behavior

**Phragmèn Lindelöf theorem**:
If $f(z) \longrightarrow a$ as $|z| \longrightarrow \infty$ along a straight line, and $f(z) \longrightarrow b$ as $|z| \longrightarrow \infty$ along another straight line, and $f(z)$ is regular and bounded in the angle between, then $a = b$ and $f(z) \longrightarrow a$ **uniformly in this angle**.

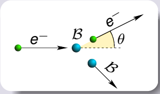
- $$\underbrace{\lim_{q^2\to-\infty} G_{E,M}(q^2)}_{\text{space-like}} = \underbrace{\lim_{q^2\to+\infty} G_{E,M}(q^2)}_{\text{time-like}}$$

- $$G_{E,M} \underset{q^2\to+\infty}{\sim} (q^2)^{-2}$$ **real**




$$\textbf{Time-like: had. helicity} = \begin{cases} 1 \Rightarrow |G_E| \\ 0 \Rightarrow |G_M| \end{cases}$$

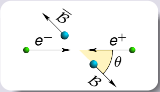
$$G_E(4M_{\mathcal{B}}^2) = G_M(4M_{\mathcal{B}}^2)$$




**Elastic scattering**

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E_e' \cos^2\frac{\theta}{2}}{4E_e^3 \sin^4\frac{\theta}{2}} \left[ G_E^2 - \tau \left( 1 + 2(1-\tau)\tan^2\frac{\theta}{2} \right) G_M^2 \right] \frac{1}{1-\tau}$$

$$\tau = \frac{q^2}{4M_{\mathcal{B}}^2}$$



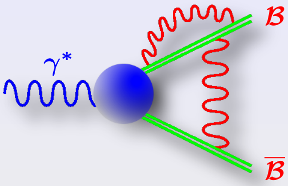

**Annihilation**

Coulomb correction

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta C}{4q^2} \left[ (1+\cos^2\theta)|G_M|^2 + \frac{1}{\tau}\sin^2\theta |G_E|^2 \right]$$

$$\beta = \sqrt{1 - \frac{1}{\tau}}$$

# The Coulomb Factor




**$p\bar{p}$ Coulomb interaction as FSI**

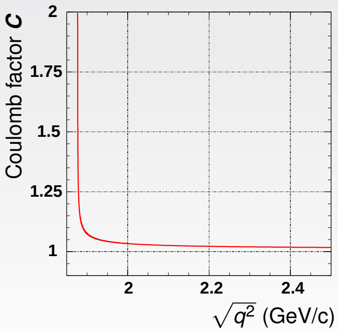
[Sommerfeld, Sakharov, Schwinger, Fadin, Khoze]

**Distorted wave approximation**

$$C = |\Psi_{\text{Coul}}(0)|^2$$

- **S-wave:** $C = \dfrac{\frac{\pi\alpha}{\beta}}{1 - \exp\left(-\frac{\pi\alpha}{\beta}\right)} \xrightarrow[\beta\to 0]{} \dfrac{\pi\alpha}{\beta}$
- **D-wave:** $C = 1$

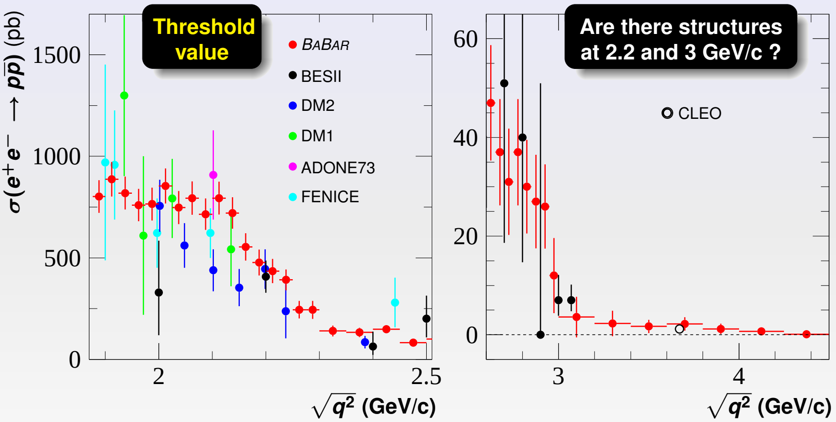
**No Coulomb factor for boson pairs (P-wave)**



# Pointlike Baryons?

R. Baldini Ferroli, SP, A. Zallo
and A. Zichichi

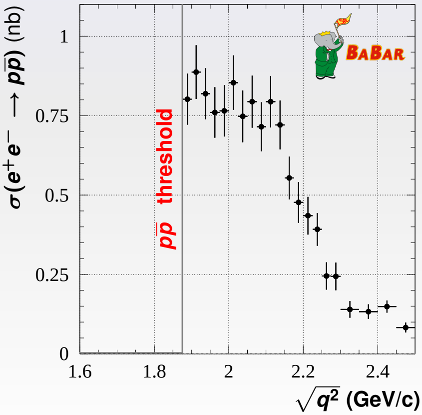
# $e^+e^- \longrightarrow p\bar{p}$: the world data sample



# $e^+e^- \longrightarrow p\bar{p}\gamma$ (ISR)
# The incredible threshold value

BABAR
PRD73, 012005

$$\sigma(e^+e^- \to p\bar{p}) = \frac{4\pi\alpha^2\beta C}{3q^2}\left[|G_M^p|^2 + \frac{2M_p^2}{q^2}|G_E^p|^2\right] \xrightarrow[\sqrt{q^2}\to 2M_p]{} \frac{\pi\alpha^2\beta C}{2M_p^2}|G^p|^2$$




At threshold
$\sigma(e^+e^- \to p\bar{p}) = 0.83 \pm 0.05$ nb

**$e^+e^- \longrightarrow p\bar{p}$ is the only endothermic process that shows this peculiarity**

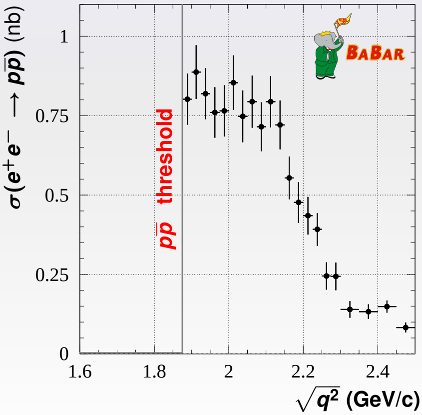
$\Downarrow$

The factor $C$ cancels the phase space

$$\beta C = \sqrt{1 - \frac{4M_p^2}{q^2}}\,C = \begin{cases} \text{finite at} \\ \text{threshold} \end{cases}$$

# $e^+e^- \longrightarrow p\bar{p}\gamma$ (ISR)
## The incredible threshold value

$$\sigma(e^+e^- \to p\bar{p}) = \frac{4\pi\alpha^2\beta C}{3q^2}\left[|G_M^p|^2 + \frac{2M_p^2}{q^2}|G_E^p|^2\right] \xrightarrow[\sqrt{q^2}\to 2M_p]{} \frac{\pi\alpha^2\beta C}{2M_p^2}|G^p|^2$$

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# Proton form factor at $q^2 = 4M_p^2$

$$\sigma(e^+e^- \to p\bar{p})(4M_p^2) = 0.83 \pm 0.05\ \text{nb}$$

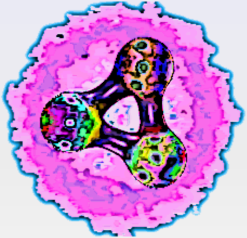
BABAR

$$\sigma(e^+e^- \to p\bar{p})(4M_p^2) = \frac{\pi^2\alpha^3}{2M_p^2}\frac{\cancel{\beta}}{\cancel{\beta}}|G^p(4M_p^2)|^2 = 0.85\,|G^p(4M_p^2)|^2\ \text{nb}$$

$$|G^p(4M_p^2)| \equiv 1$$

$$|G^p(4M_p^2)| = 0.99 \pm 0.04(\text{stat}) \pm 0.03(\text{syst})$$

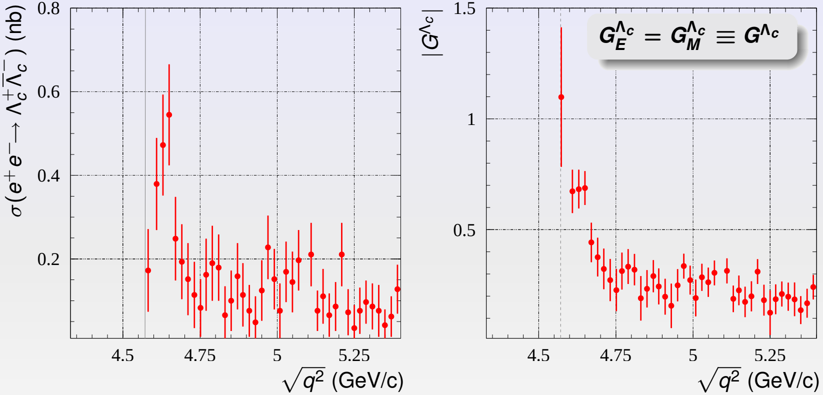
$$|G^p(4M_p^2)| \equiv 1$$



**At $q^2 = 4M_P^2$ protons behave as pointlike fermions!**

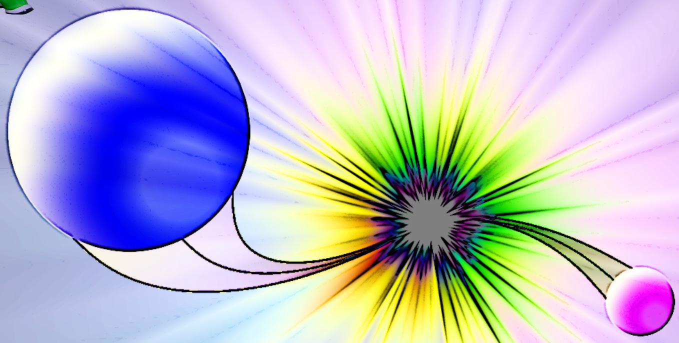
# Other charged baryon FF's at threshold

# $\Lambda_c^+$ form factor




$$|G^{\Lambda_c}(4M^2_{\Lambda_c})| = \mathbf{1.1 \pm 0.3(stat) \pm 0.4(syst)}$$

# The neutral baryons puzzle



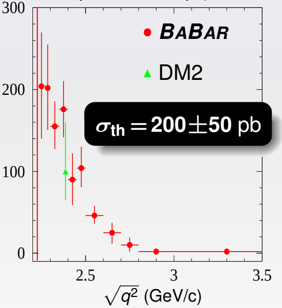
# Neutral Baryons puzzle (*BABAR*)

***BABAR***
**PRD76, 092006**

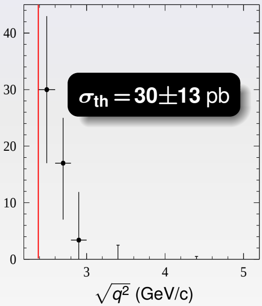
$$\sigma(e^+e^- \to \mathcal{B}^0\overline{\mathcal{B}}^0) = \frac{4\pi\alpha^2\beta C_0}{3q^2}\left[|G_M^{\mathcal{B}^0}|^2 + \frac{2M_{\mathcal{B}^0}^2}{q^2}|G_E^{\mathcal{B}^0}|^2\right] \xrightarrow[\sqrt{q^2}\to 2M_{\mathcal{B}^0}]{} \frac{\pi\alpha^2\beta}{2M_{\mathcal{B}^0}^2}|G^{\mathcal{B}^0}|^2 \to 0$$

**No Coulomb correction at hadron level: $C_0 = 1$**

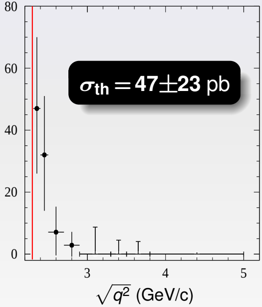
$\sigma(e^+e^- \to \Lambda\overline{\Lambda})$ (pb)




$\sigma(e^+e^- \to \Sigma^0\overline{\Sigma^0})$ (pb)



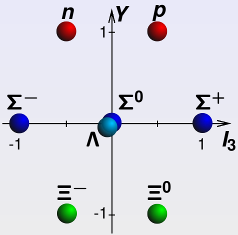

$\sigma(e^+e^- \to \Lambda\overline{\Sigma^0})$ (pb)




Like a remnant of Coulomb interactions at quark level? $\Longrightarrow$ $C_0 \propto \beta^{-1}$ as $\sqrt{q^2} \to 2M_{\mathcal{B}^0}$ $\Longrightarrow$ For any neutral baryon $\sqrt{\sigma_{\mathcal{B}^0\overline{\mathcal{B}}^0}} \propto \dfrac{|G^{\mathcal{B}^0}|}{M_{\mathcal{B}^0}}$

# Baryon octet and $U$-spin

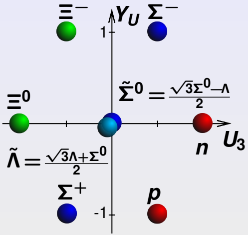
arXiv:0812.3283




$$(Y, I_3) \longrightarrow (Y_U, U_3)$$

$$U_3 = -\tfrac{1}{2} I_3 + \tfrac{3}{4} Y$$

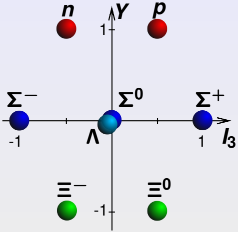
$$Y_U = -Q$$




U-spin relation: $\quad G^{\Sigma^0} - G^{\Lambda} + \frac{2}{\sqrt{3}} G^{\Lambda\Sigma^0} = 0$

$$M_{\Sigma^0}\sqrt{\sigma_{\Sigma^0\overline{\Sigma^0}}} - M_{\Lambda}\sqrt{\sigma_{\Lambda\overline{\Lambda}}} + \frac{2}{\sqrt{3}}\overline{M_{\Lambda\Sigma^0}}\sqrt{\sigma_{\Lambda\overline{\Sigma^0}}} = (-0.06 \pm 6.0) \times 10^{-4}$$

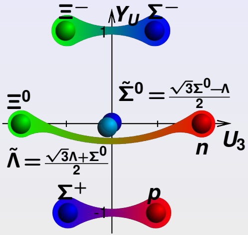
# Baryon octet and $U$-spin




$$\underline{(Y, I_3) \longrightarrow (Y_U, U_3)}$$

$$U_3 = -\tfrac{1}{2} I_3 + \tfrac{3}{4} Y$$

$$Y_U = -Q$$




<u>U-spin relation</u>: $G^{\Sigma^0} - G^{\Lambda} + \frac{2}{\sqrt{3}} G^{\Lambda\Sigma^0} = 0$

$$M_{\Sigma^0}\sqrt{\sigma_{\Sigma^0\overline{\Sigma^0}}} - M_{\Lambda}\sqrt{\sigma_{\Lambda\bar{\Lambda}}} + \frac{2}{\sqrt{3}}\overline{M_{\Lambda\Sigma^0}}\sqrt{\sigma_{\Lambda\overline{\Sigma^0}}} = (-0.06 \pm 6.0) \times 10^{-4}$$

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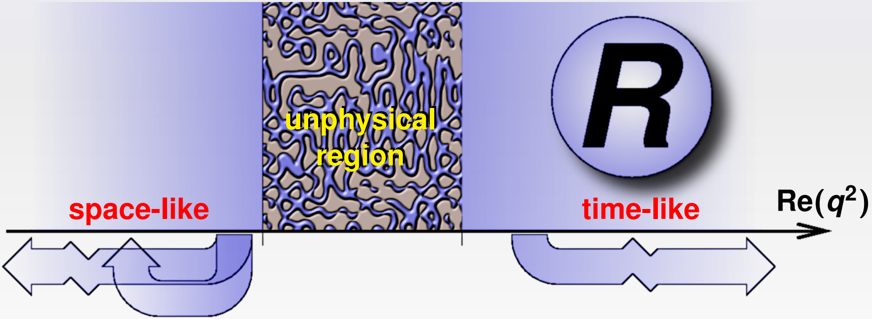
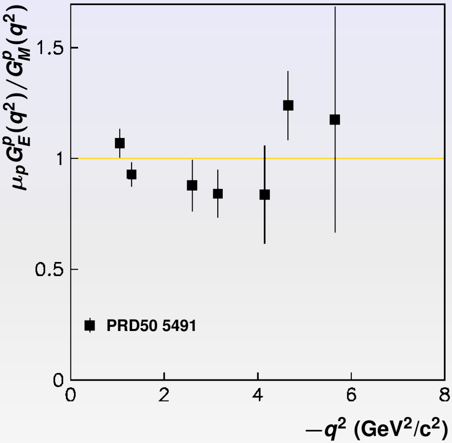
# Dispersive analysis of the ratio $R = \mu_p \frac{G_E^p}{G_M^p}$

Eur. Phys. J. A32, 421
R. Baldini, S. Pacetti and A. Zallo



# Space-like $G_E^p / G_M^p$ measurements




$$G_E^p = F_1^p + \frac{q^2}{4M_p^2} F_2^p$$

$$G_M^p = F_1^p + F_2^p$$

**Space-like**

$F_1 \,/\, \frac{q^2}{4M_p^2} F_2$ cancellation

$$\frac{G_E^p(q^2)}{G_M^p(q^2)} < 1$$

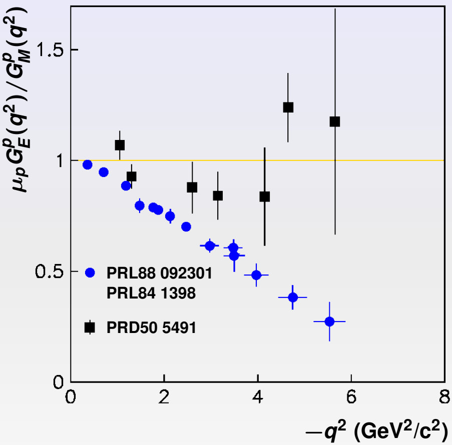
**Time-like**

$F_1 \,/\, \frac{q^2}{4M_p^2} F_2$ enhancement

$$\left|\frac{G_E^p(q^2)}{G_M^p(q^2)}\right| > 1$$

Radiative corrections of polarization technique $\ll$ Radiative corrections in Rosenbluth method

# Space-like $G_E^p / G_M^p$ measurements




$$G_E^p = F_1^p + \frac{q^2}{4M_p^2} F_2^p$$

$$G_M^p = F_1^p + F_2^p$$

**Space-like**

$F_1$ / $\frac{q^2}{4M_p^2} F_2$ **cancellation**

$$\frac{G_E^p(q^2)}{G_M^p(q^2)} < 1$$

**Time-like**

$F_1$ / $\frac{q^2}{4M_p^2} F_2$ **enhancement**

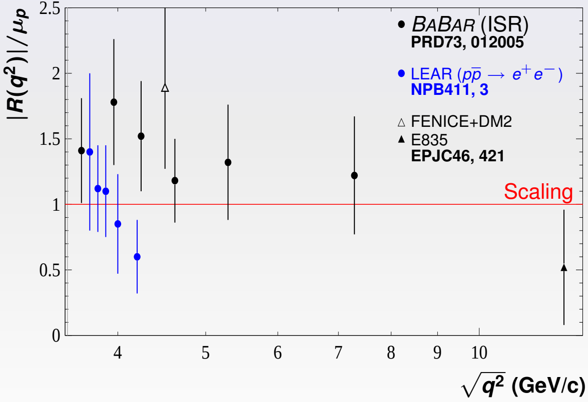
$$\left|\frac{G_E^p(q^2)}{G_M^p(q^2)}\right| > 1$$

**Radiative corrections of polarization technique** $\ll$ **Radiative corrections in Rosenbluth method**

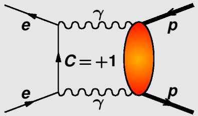
# Time-like $|G_E^p/G_M^p|$ measurements

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2\beta C}{2q^2}|G_M^p|^2\left[(1+\cos^2\theta)+\frac{4M_p^2}{q^2\mu_p^2}\sin^2\theta|R|^2\right]$$

$$R(q^2) = \mu_p \frac{G_E^p(q^2)}{G_M^p(q^2)}$$




**$\gamma\gamma$ exchange**




$\gamma\gamma$ exchange interferes with the Born term

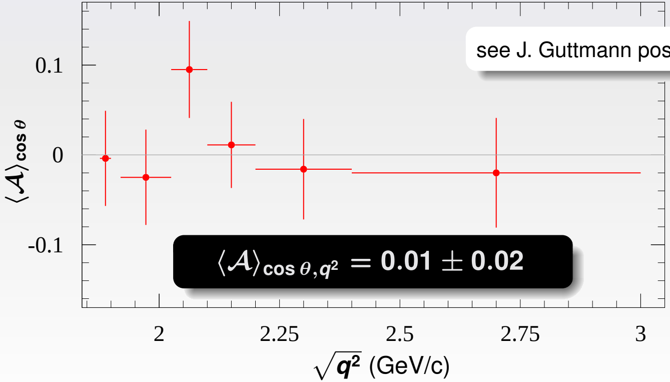
Asymmetry in angular distributions

**[PLB659, 197]**

# $\gamma\gamma$ exchange from $e^+e^- \to p\bar{p}\gamma$ BABAR data

E. Tomasi-Gustafsson, E. A. Kuraev, S. Bakmaev, SP
PLB659, 197

$$\mathcal{A}(\cos\theta, q^2) = \frac{\frac{d\sigma}{d\Omega}(\cos\theta, q^2) - \frac{d\sigma}{d\Omega}(-\cos\theta, q^2)}{\frac{d\sigma}{d\Omega}(\cos\theta, q^2) + \frac{d\sigma}{d\Omega}(-\cos\theta, q^2)}$$



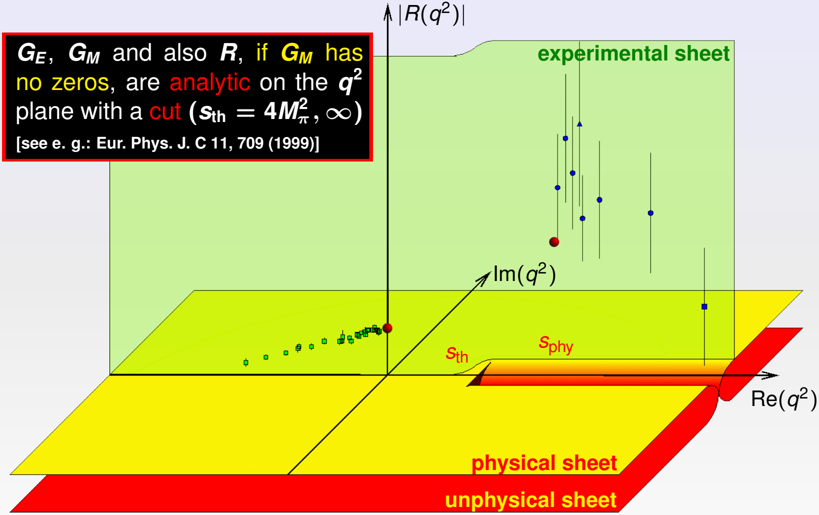

see J. Guttmann poster

$$\langle\mathcal{A}\rangle_{\cos\theta, q^2} = 0.01 \pm 0.02$$

# $R(q^2)$ in the complex plane

$G_E$, $G_M$ and also $R$, if $G_M$ has no zeros, are analytic on the $q^2$ plane with a cut $(s_{\rm th} = 4M_\pi^2, \infty)$

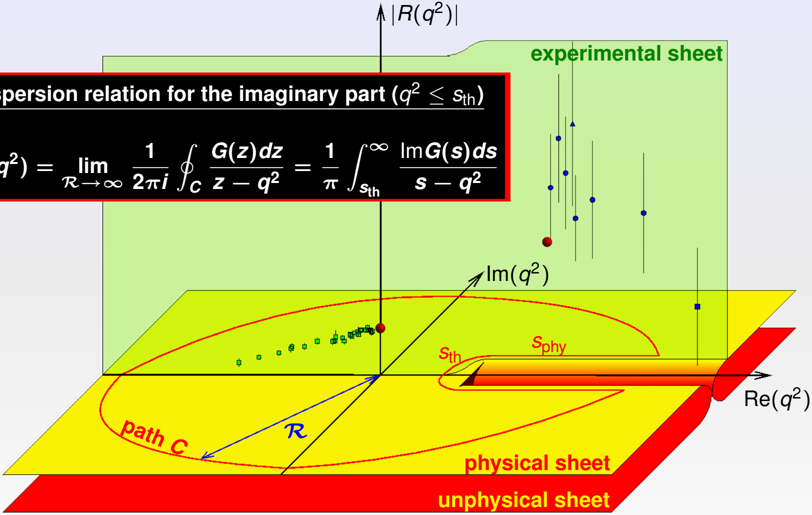
[see e. g.: Eur. Phys. J. C 11, 709 (1999)]



# $R(q^2)$ in the complex plane

**Dispersion relation for the imaginary part ($q^2 \leq s_{\text{th}}$)**

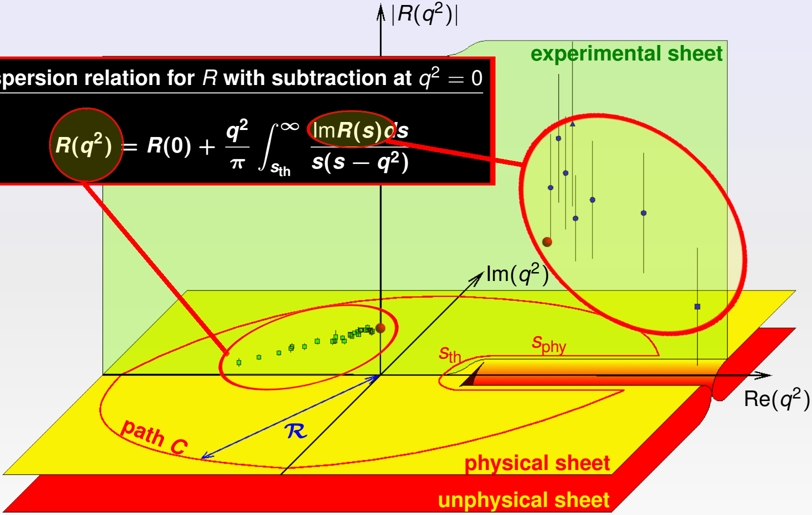
$$G(q^2) = \lim_{\mathcal{R}\to\infty} \frac{1}{2\pi i} \oint_C \frac{G(z)dz}{z - q^2} = \frac{1}{\pi} \int_{s_{\text{th}}}^{\infty} \frac{\mathrm{Im}G(s)ds}{s - q^2}$$



# $R(q^2)$ in the complex plane

**Dispersion relation for $R$ with subtraction at $q^2 = 0$**

$$R(q^2) = R(0) + \frac{q^2}{\pi}\int_{s_{th}}^{\infty} \frac{\mathrm{Im}R(s)\,ds}{s(s-q^2)}$$



# $R(q^2)$

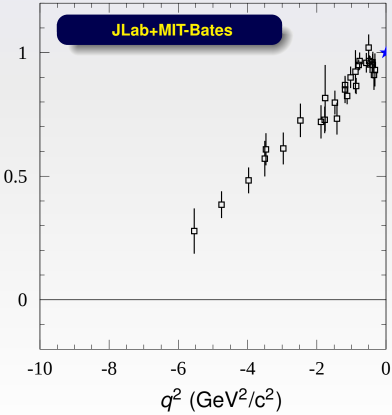


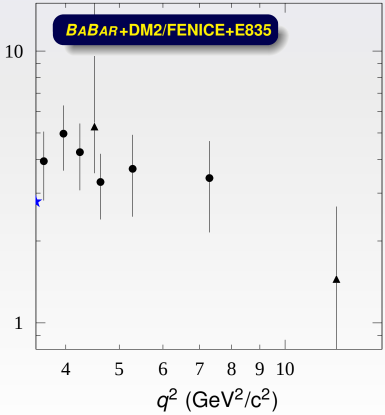
$$R(q^2) = R(0) + \frac{q^2}{\pi}\int_{4M_\pi^2}^{\infty} \frac{\mathrm{Im}R(s)}{s(s-q^2)}ds$$

Re$q^2$

$R(q^2)$ space-like

$|R(q^2)|$ time-like

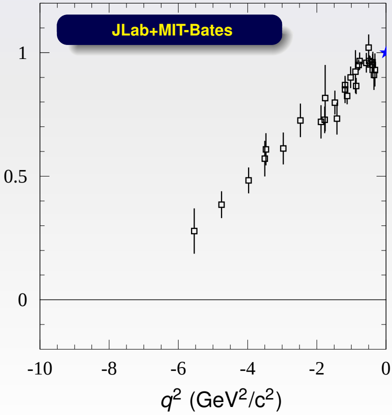


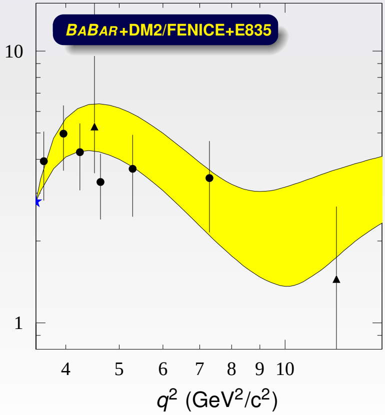
# $R(q^2)$

$$R(q^2) = R(0) + \frac{q^2}{\pi}\int_{4M_\pi^2}^{\infty} \frac{\mathrm{Im}R(s)}{s(s-q^2)}ds$$


$R(q^2)$ space-like




$|R(q^2)|$ time-like

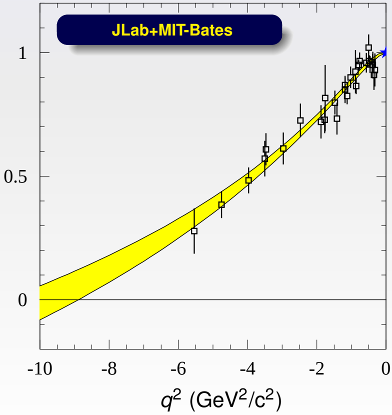


# $R(q^2)$

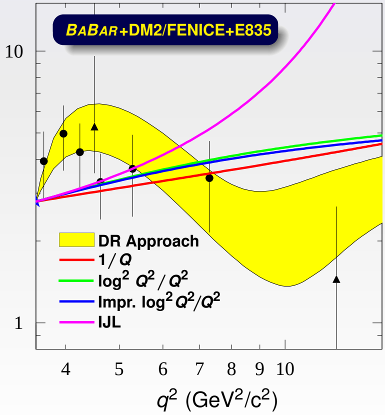
EPJA32 421

$$R(q^2) = R(0) + \frac{q^2}{\pi}\int_{4M_\pi^2}^{\infty} \frac{\mathrm{Im}R(s)}{s(s-q^2)}\,ds$$


$R(q^2)$ space-like




$|R(q^2)|$ time-like

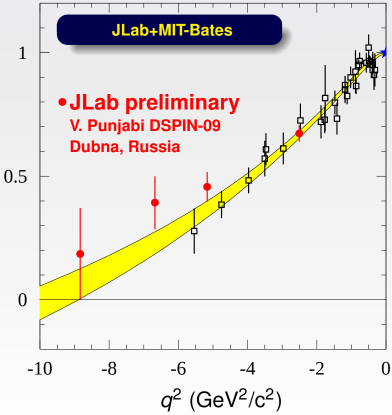


# $R(q^2)$

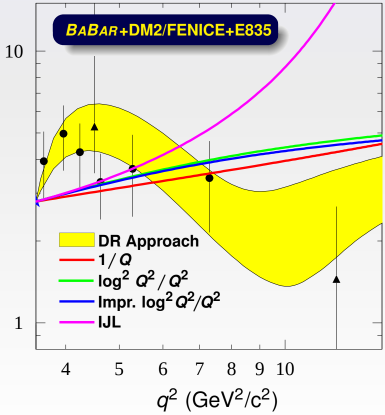


$$R(q^2) = R(0) + \frac{q^2}{\pi}\int_{4M_\pi^2}^{\infty}\frac{\mathrm{Im}R(s)}{s(s-q^2)}ds$$

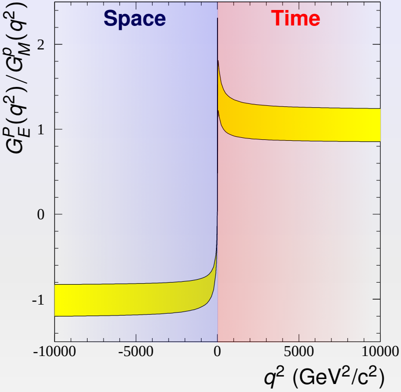

**$R(q^2)$ space-like**

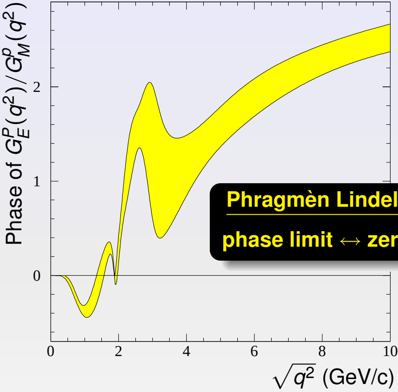



**$|R(q^2)|$ time-like**



# Asymptotic $G_E^P(q^2)/G_M^p(q^2)$ and phase




**Phragmèn Lindelöf**

**phase limit $\leftrightarrow$ zeros**

**pQCD prediction**

$$\frac{G_E^p(q^2)}{G_M^p(q^2)} \xrightarrow[|q^2|\to\infty]{} -1$$

**Phase from DR**

$$\phi(q^2) = -\frac{\sqrt{q^2-s_0}}{\pi} \, \mathrm{Pr} \int_{s_0}^{\infty} \frac{\ln|R(s)|ds}{\sqrt{s-s_0}(s-q^2)}$$

# Instead of Conclusions. . .

## Highlights

- Coulomb correction for $\mathcal{B}\overline{\mathcal{B}}$
  - Charged baryons **as pointlike fermions**
  - Puzzling cross sections at threshold for neutral baryons
- Time + space-like data for $G_E^p/G_M^p$ predict:
  - a **space-like zero**, also from time-like phase
  - the space-like limit $\boldsymbol{G_E^p/G_M^p \to -1}$

## Expectations

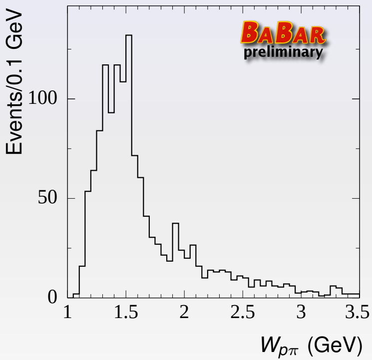
- Theoretical space-like **and time-like** interpretations
- New **polarized** and unpolarized, space and time data: BESIII, VEPP2000, Belle2, Panda (M. Sudoł), SuperB (?)

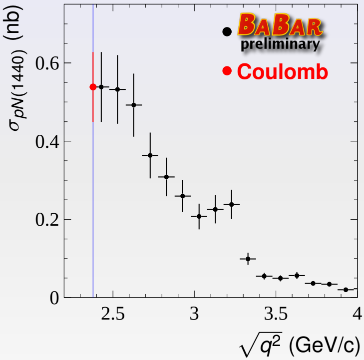
# BACK-UP SLIDES

# $e^+e^- \to p\overline{N}(1440) + \overline{p}N(1440)$



$$\sigma^{\text{Coulomb}} = \frac{16\pi^2\alpha^3 M_p^{3/2} M_{N(1440)}^{3/2}}{(M_p + M_{N(1440)})^5} \left|G^{pN(1440)}\right|^2 = \left|G^{pN(1440)}\right|^2 \times 0.49 \text{ nb}$$




$$\left|G^{pN(1440)}\right| = 1.04 \pm 0.09$$

# DR approach: advantages and drawbacks

## Advantages

- DR's are based on unitarity and analyticity $\Rightarrow$ **model-independent approach**
- DR's relate data from different processes in different energy regions

$$\left[\begin{array}{c}\text{space-like}\\ \text{form factor}\\ e\mathcal{B}\to e\mathcal{B}\end{array}\right] = \int \left[\begin{array}{c}\text{Im(form factor) or ln}|\text{form factor}|\\ \text{over the time-like cut } (s_{\text{th}},\infty)\\ e^+e^-\to \mathcal{B}\overline{\mathcal{B}} + \text{theory}\end{array}\right]$$

- Normalizations and theoretical constraints can be directly implemented
- Form factors can be computed in the whole $q^2$-complex plane

## Drawbacks

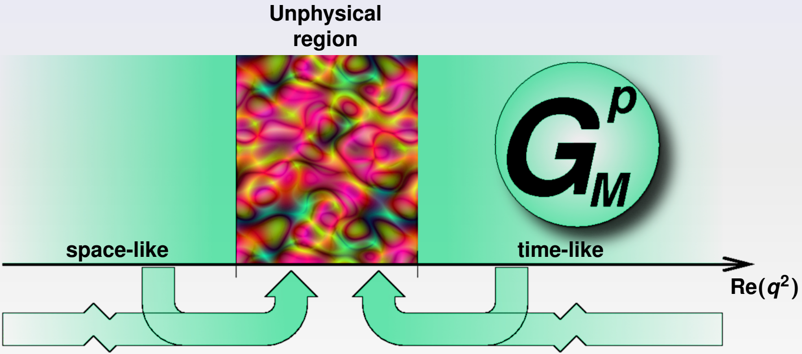
- Very long range integration

Even though pQCD provides power rules nobody knows at which energy the form factors start to follow these behaviors

Subtracted dispersion relations help in making the approach as less as possible dependent on the asymptotic behavior

- No data in the unphysical region

# Proton magnetic form factor with unphysical-contribution suppression



# Dispersion relations and sum rules

**Geshkenbeĭn, Ioffe, Shifman Yad. Fiz. 20, 128 (1974)**

- **DR's connect space and time values of a form factor $G(q^2)$**

$$G(q^2) = \frac{1}{\pi}\int_{s_{\rm th}}^{\infty} \frac{{\rm Im}G(s)ds}{s-q^2}$$


**Drawbacks**

- **The imaginary part is not experimentally accessible**
- **There are no data in the unhysical region** $[s_{\rm th}, s_{\rm phy}]$
- **We need to know the asymptotic behavior**

- **They applied the DR for the imaginary part to the function**

$$\phi(z) = f(z)\frac{\ln G(z)}{z\sqrt{s_{\rm th}-z}} \quad \text{with} \quad \int_0^{s_{\rm phy}} f^2(z)dz << 1$$

**Advantages**

- **The DR integral contains the modulus** $|G(s)|$
- **The unhysical region contribution is suppressed**

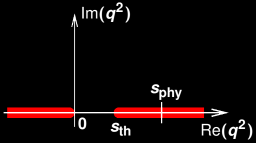
**Drawback**

- **Zeros of** $G(z)$ **are poles for** $\phi(z)$

# Attenuation of the unphysical region

**Strategy**

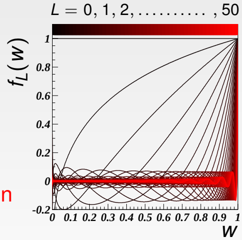
- Use the function $\phi(z) = f(z)\dfrac{\ln G(z)}{z\sqrt{s_{\rm th} - z}}$
- $f(z)$, is analytic with the cut $(-\infty, 0)$
- $f(z) = f_L(w) = \sum_{l=0}^{L} \dfrac{2l+1}{(L+1)^2} P_l(1-2w),\ w = \dfrac{\sqrt{s_{\rm phy}} - \sqrt{z}}{\sqrt{s_{\rm phy}} + \sqrt{z}}$




- This function, with $f_L(0)=1$, minimizes:

$$\int_0^1 f_L^2(w)\,dw$$

and suppresses the contribution in the unphysical region



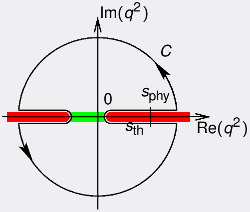
# Attenuated DR and sum rule

## New DR with variable suppressed region $[0, s_{\rm phy}]$ $[G(q^2)$ has no zeros$]$

$$\boxed{\oint_C \phi(z)dz = 0}$$

$$\Downarrow$$

$$\underbrace{-\int_{-\infty}^{0} \frac{\mathrm{Im}[f(t)] \ln G(t)}{t\sqrt{s_{\rm th} - t}} dt}_{\text{Space-like}} = \underbrace{\int_{s_{\rm th}}^{\infty} \frac{f(s) \ln |G(s)|}{s\sqrt{s - s_{\rm th}}} ds}_{\text{Time-like}}$$



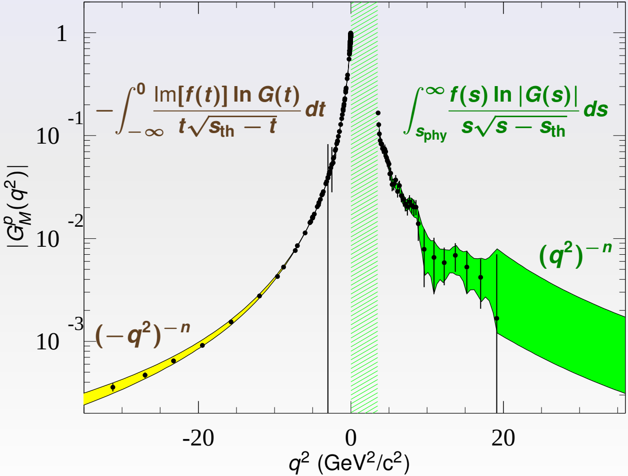

## Convergence relation to test asymptotic power behaviour of $G_M^p$

$$\underbrace{-\int_{-\infty}^{0} \frac{\mathrm{Im}[f(t)] \ln G(t)}{t\sqrt{s_{\rm th} - t}} dt}_{\text{Space-like data + } (-t)^{-n}} = \int_{s_{\rm th}}^{\infty} \frac{f(s) \ln |G(s)|}{s\sqrt{s - s_{\rm th}}} ds \approx \underbrace{\boxed{\int_{s_{\rm phy}}^{\infty} \frac{f(s) \ln |G(s)|}{s\sqrt{s - s_{\rm th}}} ds}}_{\text{Time-like data + } s^{-n}}$$

***n* is the free parameter**

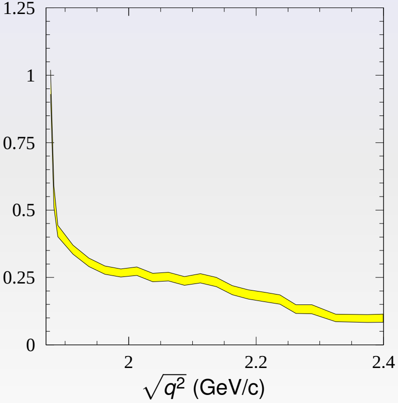
# Sum rule: result

$$G_M^p(q^2) \underset{|q^2|\to\infty}{\propto} (q^2)^{-(2.27\pm0.36)}$$

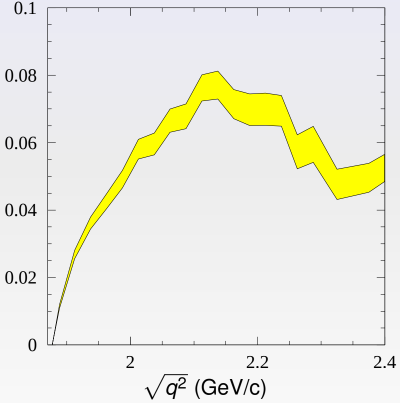


# Phases from DR: $|B_S^p(q^2)|$ and $|B_D^p(q^2)|$

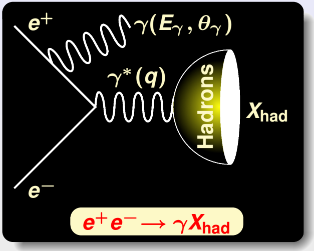
$$|B_S^p(q^2)| = \frac{|2\sqrt{\tau}G_M^p + G_E^p|}{3}$$




$$|B_D^p(q^2)| = \frac{|\sqrt{\tau}G_M^p - G_E^p|}{3}$$



# Initial State Radiation




$$\frac{d^2\sigma}{dE_\gamma\, d\theta_\gamma} = W(E_\gamma, \theta_\gamma) \cdot \sigma_{e^+e^- \to X_{\text{had}}}(s)$$

- $s = q^2$, $q \,..\, X_{\text{had}}$ momentum
- $E_\gamma$ ............ CM $\gamma$ energy
- $\theta_\gamma$ ... CM $\gamma$ scattering angle
- $E_{\text{CM}}$ ...... CM $e^+e^-$ energy

## Radiator function in Born approximation

$$W(E_\gamma, \theta_\gamma) = \frac{\alpha}{\pi x} \left( \frac{2 - 2x + x^2}{\sin^2 \theta_\gamma} \right), \qquad x = \frac{E_\gamma}{2E_{\text{CM}}}$$

$$\theta_\gamma \gg \frac{m_e}{E_{\text{CM}}}$$

**For $20^o < \theta_\gamma < 160^o$ ISR angular acceptance $\sim 17\%$**

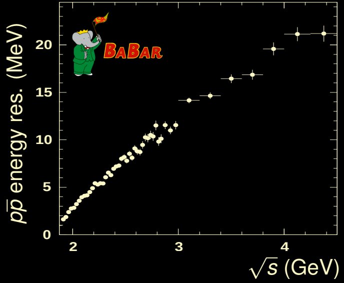
# ISR versus CM scan

## Advantages

- All energies ($q^2$) at the same time

  $\Downarrow$

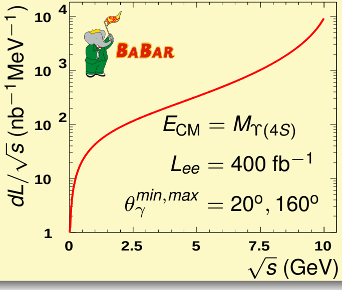
  Better control on systematics
  (e.g. greatly reduced point to point)
- Detected ISR $\Longrightarrow$ full $X_{\rm had}$ ang. coverage
- CM boost $\Longrightarrow$ $\begin{cases} \text{at threshold } \epsilon \neq \mathbf{0} \\ \text{energy res. } \sim \mathbf{1\ MeV} \end{cases}$




## Drawbacks

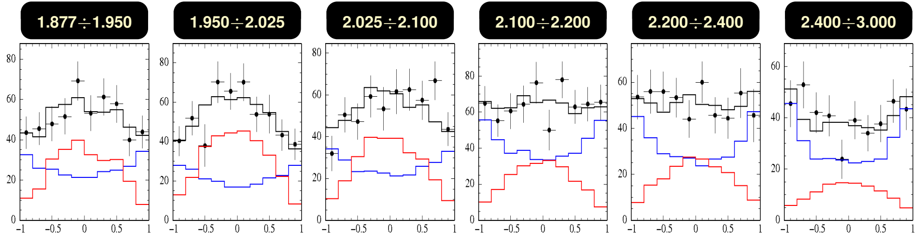
- ISR $\mathcal{L} \propto$ energy bin width
- More background: non-ISR events

$$\frac{dL}{d\sqrt{s}} = \frac{2\sqrt{s}}{E^2_{\rm CM}} L_{ee} \int_{\cos\theta^{\rm min}_\gamma}^{\cos\theta^{\rm max}_\gamma} d\cos\theta_\gamma \, W(E_\gamma, \theta_\gamma)$$



# $e^+e^- \longrightarrow p\bar{p}$ angular distribution (*BaBar*)

## $\cos\theta_p$ distributions form threshold up to 3 GeV [intervals in $E_{CM} \equiv q$ (GeV)]

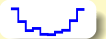



Events/0.2 vs. $\cos\theta_p$

$$\frac{d\sigma}{d\cos\theta_p} = A\left[H_E(\cos\theta_p, q^2)\left|\frac{G_E^p(q^2)}{G_M^p(q^2)}\right|^2 + H_M(\cos\theta_p, q^2)\right]$$

$H_E$ and $H_M$ from MC

**Histograms show contributions from**

- $G_E$ 
- $G_M$ 

**At low $q$**

$\sin^2\theta_p > 1 + \cos^2\theta_p$

$\Rightarrow$

**First observation!**

$|G_E^p| > |G_M^p|$

**At higher $q$, $|G_E^p| \longrightarrow |G_M^p|$**

# Parameterization and constraints

**The imaginary part of $R$ is parameterized by two series of orthogonal polynomials $T_i(x)$**

$$\mathrm{Im}R(q^2) \equiv I(q^2) = \begin{cases} \sum_i C_i T_i(x) & x = \dfrac{2q^2 - s_{\rm phy} - s_{\rm th}}{s_{\rm phy} - s_{\rm th}} \quad s_{\rm th} \leq q^2 \leq s_{\rm phy} \\ \sum_j D_j T_j(x') & x' = \dfrac{2s_{\rm phy}}{q^2} - 1 \quad q^2 > s_{\rm phy} \end{cases}$$

$$s_{\rm th} = 4M_\pi^2$$
$$s_{\rm phy} = 4M_N^2$$

**Theoretical conditions on $ImR(q^2)$**

- $R(4M_\pi^2)$ is real $\Longrightarrow I(4M_\pi^2) = 0$
- $R(4M_N^2)$ is real $\Longrightarrow I(4M_N^2) = 0$
- $R(\infty)$ is real $\Longrightarrow I(\infty) = 0$

**Theoretical conditions on $R(q^2)$**

- Continuity at $q^2 = 4M_\pi^2$
- $R(4M_N^2)$ is real and $\mathrm{Re}R(4M_N^2) = \mu_p$

**Experimental conditions on $R(q^2)$ and $|R(q^2)|$**

- Space-like region ($q^2 < 0$) data for $R$ from TJNAF and MIT-Bates
- Time-like region ($q^2 \geq 4M_N^2$) data for $|R|$ from FENICE+DM2, *BaBar*, E835 and LEAR

# Asymptotic value and space-like zero



- **Real asymptotic values for $R$**

$$R_{BABAR}(\infty) = -(1.0 \pm 0.2)\mu_p$$

**BABAR is in agreement with the scaling law $|G_E(q^2)| \simeq |G_M(q^2)|$ as $q^2 \to \infty$**

- **Asymptotic behaviour of $F_2/F_1$**

$$\lim_{q^2\to\infty} \frac{q^2}{4M_N^2}\left|\frac{F_2}{F_1}\right| = \left|\frac{R(\infty)}{\mu_p} - 1\right| = 2.0 \pm 0.2 \text{ (BABAR)}$$

$$\left|\frac{F_2}{F_1}\right| \underset{q^2\to\infty}{\propto} \frac{1}{q^2}$$

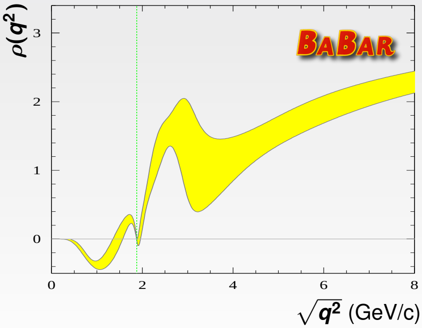
- **Space-like zero**

$$t_0^{BABAR} = (-10 \pm 1) \ \text{GeV}^2/\text{c}^2$$

- **Phragmèn-Lindelöff theorem**

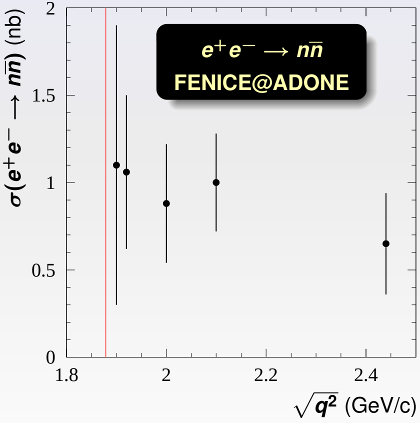
$$\rho(q^2) \xrightarrow[q^2\to\infty]{} \pi \text{ with}$$

$$R(q^2) = |R(q^2)|e^{i\rho(q^2)}$$



# $U$-spin prediction for neutrons at threshold

$$\sigma(e^+e^- \to n\bar{n}) = \left(3\sqrt{\sigma_{\Lambda\bar{\Lambda}}}M_\Lambda - \sqrt{\sigma_{\Sigma^0\bar{\Sigma}^0}}M_\Sigma\right)^2 \frac{1}{4M_n^2} = 0.5 \pm 0.2 \text{ nb}$$

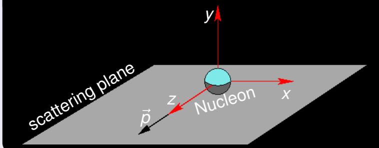


# Polarization formulae in the time-like region

The ratio $R(q^2)$ is complex for $q^2 \geq s_{\text{th}}$

$$R(q^2) = \mu_p \frac{G_E^p(q^2)}{G_M^p(q^2)} = |R(q^2)|e^{i\rho(q^2)}$$

The polarization depends on the phase $\rho$




$$\mathcal{P}_y = -\frac{\sin(2\theta)|R|\sin(\rho)}{D\sqrt{\tau}}$$

Does not depend on $P_e$

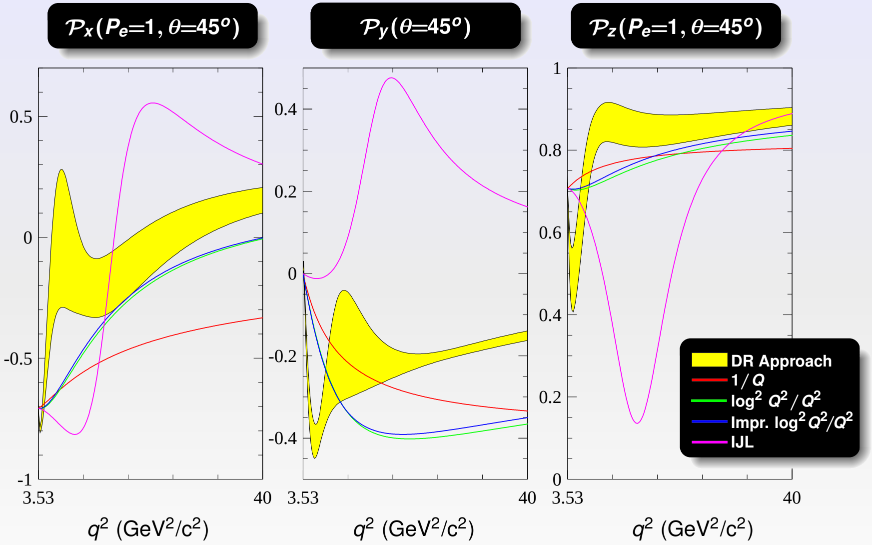
$$\mathcal{P}_x = -P_e \frac{2\sin(2\theta)|R|\cos(\rho)}{D\sqrt{\tau}}$$

$$\mathcal{P}_z = P_e \frac{2\cos(\theta)}{D}$$

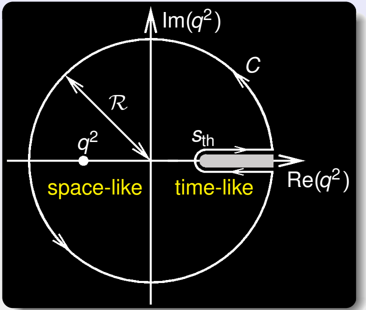
Does not depend on $\rho$

$$D = \frac{1 + \cos^2\theta + \frac{1}{\tau}|R|^2 \sin^2\theta}{\mu_p} \qquad \tau = \frac{q^2}{4M_N^2} \qquad P_e = \text{electron polarization}$$

# Single Polarization



# Dispersion relations




- The form factors are **analytic** on the $q^2$-plane with a **multiple cut** $(s_{\rm th} = 4M_\pi^2\,,\,\infty)$
- **Dispersion relation for the imaginary part** $(q^2 < 0)$

$$G(q^2) = \lim_{\mathcal{R}\to\infty} \frac{1}{2\pi i}\oint_C \frac{G(z)dz}{z - q^2} = \frac{1}{\pi}\int_{s_{\rm th}}^{\infty} \frac{{\rm Im}G(s)ds}{s - q^2}$$

- **Dispersion relation for the logarithm** $(q^2 < 0)$

B.V. Geshkenbein, Yad. Fiz. 9 (1969) 1232.

$$\ln G(q^2) = \frac{\sqrt{s_{\rm th} - q^2}}{\pi}\int_{s_{\rm th}}^{\infty} \frac{\ln|G(s)|ds}{(s - q^2)\sqrt{s - s_{\rm th}}}$$

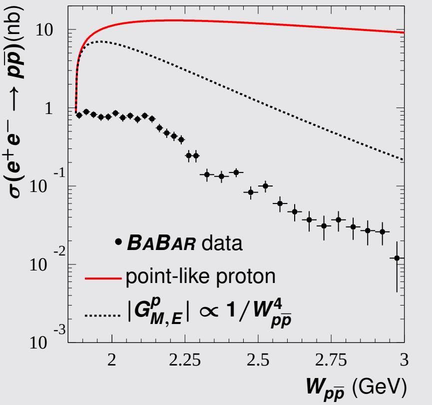
## Experimental inputs

- Space-like data on the real values of FF's from: $e^-\mathcal{B} \longrightarrow e^-\mathcal{B}$ and $e^{-\uparrow}\mathcal{B} \longrightarrow e^-\mathcal{B}^{\uparrow}$, with polarization
- Time-like data on moduli of FF's from: $e^+e^- \longrightarrow \mathcal{B}\overline{\mathcal{B}}$
- Time-like data on $G_E$-$G_M$ relative phase from: $e^+e^- \longrightarrow \mathcal{B}^{\uparrow}\overline{\mathcal{B}}$ (pol.)

## Theoretical ingredients

- Analyticity $\Rightarrow$ dispersion relations
- Normalization and threshold values
- Asymptotic behavior $\Rightarrow$ super-convergence relations

# . . . a non trivial nucleon structure




## Simple models for FF's

- **Point-like proton (red curve)**

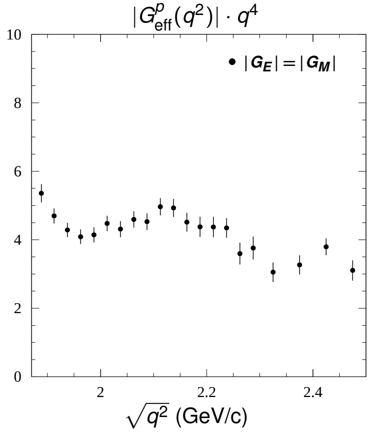
$$G_E^p = G_M^p \equiv 1$$

- **Counting rule (dashed curve)**

$$|G^p_{M,E}| \propto 1/W^4_{p\bar{p}}$$
$$\Downarrow$$
$$\sigma(e^+e^- \to p\bar{p}) \propto 1/W^{10}_{p\bar{p}}$$

**Additional factors related to $\beta$ and non-trivially structured electric and magnetic FF's must be included to reproduce the flat behavior of the data**

# $|G_E^p(q^2)|$ and $|G_M^p(q^2)|$ from $\sigma_{p\bar{p}}$ and DR

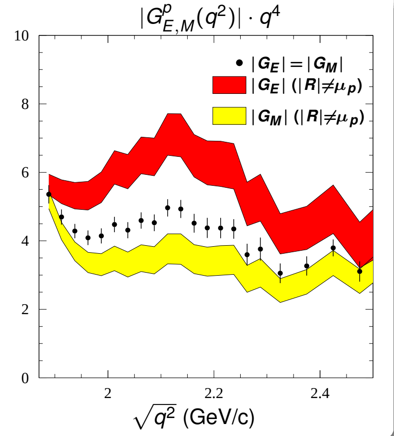



$$|G_{\rm eff}(q^2)|^2 = \frac{\sigma_{p\bar{p}}(q^2)}{\frac{4\pi\alpha^2\beta C}{3s}}\left(1+\frac{1}{2\tau}\right)^{-1}$$

- **Usually what is extracted from the cross section $\sigma(e^+e^- \longrightarrow p\bar{p})$ is the effective time-like form factor $|G_{\rm eff}^p|$ obtained assuming $|G_E^p| = |G_M^p|$ i.e. $|R| = \mu_p$**
- **Using our parametrization for $R$ and the BABAR data on $\sigma(e^+e^- \longleftrightarrow p\bar{p})$, $|G_E^p|$ and $|G_M^p|$ may be disentangled**

# $|G_E^p(q^2)|$ and $|G_M^p(q^2)|$ from $\sigma_{p\bar{p}}$ and DR






$$|G_M(q^2)|^2 = \frac{\sigma_{p\bar{p}}(q^2)}{\frac{4\pi\alpha^2\beta C}{3s}} \left(1 + \frac{|R(q^2)|}{2\mu_p\tau}\right)^{-1}$$

- **Usually what is extracted from the cross section $\sigma(e^+e^- \longrightarrow p\bar{p})$ is the effective time-like form factor $|G^p_{\text{eff}}|$ obtained assuming $|G^p_E| = |G^p_M|$ i.e. $|R| = \mu_p$**
- **Using our parametrization for $R$ and the BABAR data on $\sigma(e^+e^- \leftrightarrow p\bar{p})$, $|G^p_E|$ and $|G^p_M|$ may be disentangled**